Qualifying Exam Syllabus

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1 Main Topic

The exam topic is based on the paper "Point-like Bounding Chains in Open Gromov-Witten Theory" by Jake Solomon, Sara Tukachinsky. The presentation will be a discussion of Gromov compactness and the construction of the moduli space of J-holomorphic maps, an overview of progress in the development of open Gromov-Witten invariants, followed by proofs of Theorems 1-6 of the paper.

2 Prerequisites

2.1 Symplectic Geometry

- Linear symplectic geometry, symplectic vector spaces, symplectic linear group, symplectic vector bundles
- Symplectic manifolds, Hamiltonian flows, Darboux's theorem, Moser's Theorem, cotangent bundle, Weinstein's Lagrangian neighborhood theorem, Kähler manifolds, almost complex structures, integrability, contact manifolds
- Symplectic group actions, moment maps, symplectic quotient, symplectic fibrations

References: McDuff-Salamon, *Symplectic Topology*, Ch. 2-6

2.2 Complex Geometry

- Complex manifolds, holomorphic vector bundles, Dolbeaut cohomology, line bundles and divisors, blow ups
- Kähler manifolds, Kähler identities, Hodge decomposition
- Hermitian vector bundles, Serre duality
- Connections, curvature, Chern classes, definition, computations
- Kodaira vanishing (required) and Kodaira embedding theorem
- Riemann surfaces, branched coverings, Riemann-Hurwitz theorem (required), genus-degree theorem, Riemann-Roch theorem, uniformization, projectivity of smooth compact Riemann surfaces

References: Huybrechts, *Complex Geometry*, Ch. 2-5

2.3 J-holomorphic Curves

- J-holomorphic curves, definition and properties, energy, positivity of intersection.
- Transversality of moduli space of simple J-holomorphic curves, linearization of Cauchy-Riemann equation, usage of Sard's theorem and Implicit function theorem for Banach manifolds.
- Moduli space of stable J-holomorphic maps
- Definition of closed Gromov-Witten invariants for semi-positive symplectic manifolds, pseudocycles, criterions for regular almost complex structure, Gromov-Witten axioms, WDVV recursion
- Quantum cohomology, definition and properties, computation for projective space
- Hamiltonian Floer theory, definition, invariance under choices of Hamiltonian and almost complex structure, PSS isomorphism

References: McDuff-Salamon, *J-holomorphic Curves and Symplectic Topology*, Ch. 2-7, 11-12